United States
Department of
Agriculture

Forest Service

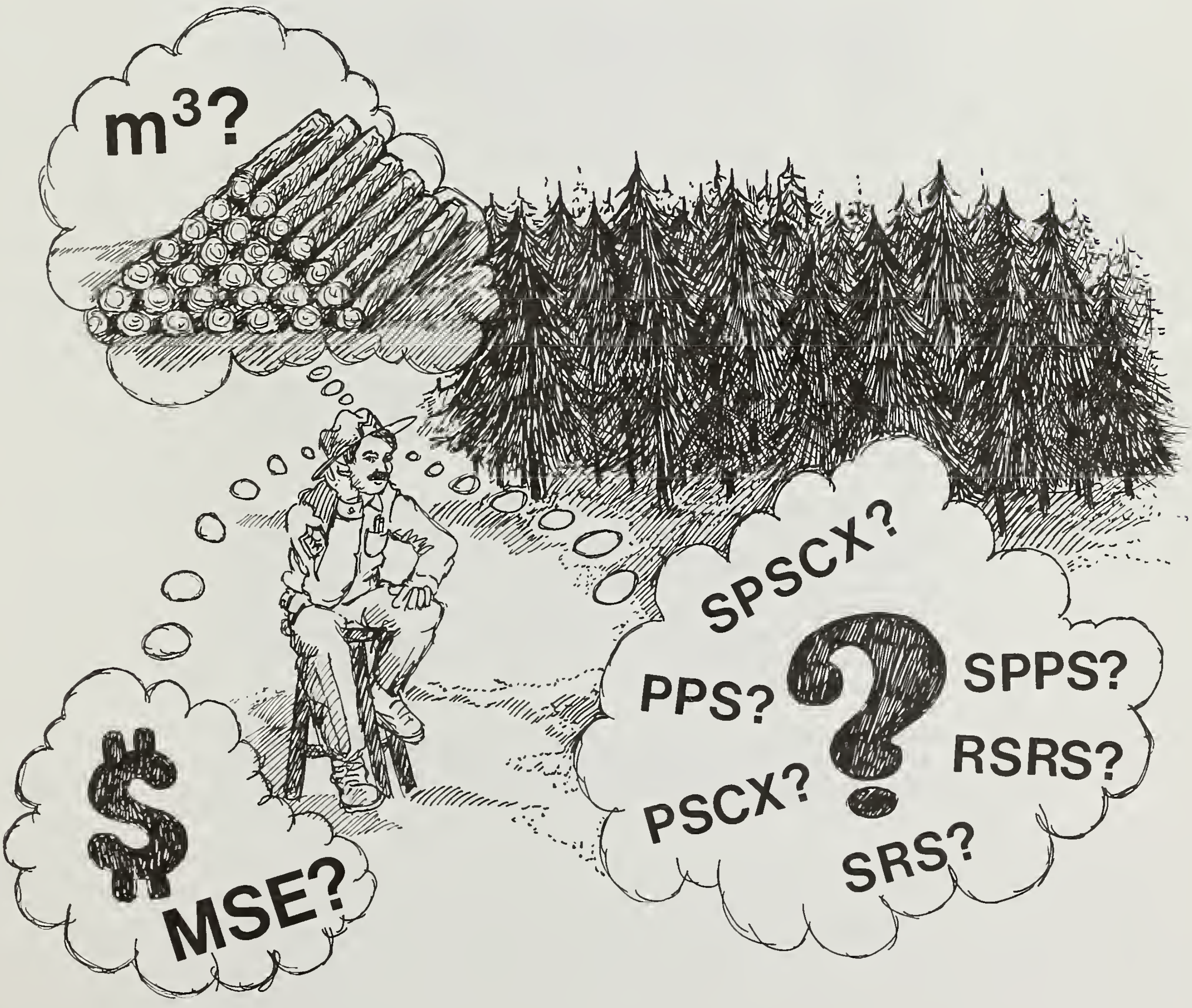
**Rocky Mountain
Forest and Range
Experiment Station**

Fort Collins,
Colorado 80526

**Research Paper
RM-291**

Model-Dependent and Design-Dependent Sampling Procedures— A Simulation Study

H. T. Schreuder, H. G. Li, and G. B. Wood



Abstract

Six sample selection schemes with seven estimators were studied in a simulation study drawing samples of 20 units from eight forestry populations. Sampling strategies were compared on the basis of bias both of the estimate and its variance, mean square error, and coverage of confidence intervals. Although there is no consistently best sampling strategy, stratified sampling with probabilities proportional to size (spps), stratified sampling proportional to the cumulated values of x (spscx), and purposive sampling from the cumulated x-values (pscx) with the weighted regression estimator $\hat{YWR}$ performed well for most situations with most of the criteria. However, pscx showed evidence of estimation bias in several instances. No reliable variance estimator was found for most sampling strategies, but the bootstrap variance estimates with spps tended to have relatively small bias ($\leq$8.58% bias for all populations). For spps sampling, use of classical and simple jackknife variance estimates but not the simple bootstrap estimates yielded reliable coverage probabilities in most populations. For many sampling strategies, variance estimation bias of up to 20% may have to be accepted. The jackknife variance estimator always overestimates the true variance. It is clear that new, reliable variance estimators may need to be derived for many sampling strategies.

Schreuder, H. T.; Li, H. G.; Wood, G. B. 1990. Model-dependent and design-dependent sampling procedures—a simulation study. Res. Pap. RM-291. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station. 19 p.

Six sample selection schemes with seven estimators were studied in a simulation study drawing samples of 20 units from eight forestry populations. Although there is no consistently best sampling strategy, stratified sampling with probabilities proportional to size (spps), stratified sampling proportional to the cumulated values of x (spscx), and purposive sampling from the cumulated x-values (pscx) with the weighted regression estimator $\hat{YWR}$ performed well for most situations with most of the criteria.

Keywords: Best sampling strategy, simulation study, model-dependent sampling procedures, design-dependent sampling procedures, jackknife estimators, bootstrap estimators, forest populations

USDA Forest Service
Research Paper RM-291

March 1990

Model-Dependent and Design-Dependent Sampling Procedures—A Simulation Study

H. T. Schreuder, Project Leader
Rocky Mountain Forest and Range Experiment Station[1]

H. G. Li, Postdoctoral Fellow
Statistics Department, Colorado State University

G. B. Wood, Senior Lecturer
Department of Forestry
Australian National University

[1]*Headquarters is in Fort Collins, in cooperation with Colorado State University.*

Contents

Model-Dependent and Design-Dependent Sampling Procedures—A Simulation Study

H. T. Schreuder, H. G. Li, and G. B. Wood

Management Implications

Inventories are expensive but are usually necessary to achieve management objectives. Some new sampling strategies, which include both design and weighted regression estimators, are shown to be highly efficient in estimating parameters of interest. It is shown that model-based but probabilistic procedures do as well or better than purposive model-based procedures. Variance estimation for such complex strategies is not yet satisfying but more precise estimates can be generated even with current designs with little or no cost to the forest manager.

Introduction

Traditional approaches to sampling such as presented in Cochran (1977) assume that sample units in surveys are selected by a design-dependent (DD) technique, such as stratified sampling, probability proportional to size (pps) sampling, or simple random sampling (SRS). If a linear model can be assumed between the variable of interest and a covariate, model-based (MB) (or more descriptively model-dependent (MD)) sampling may yield more efficient but perhaps more biased estimates and the estimate of the variance may be unreliable. The purpose of this simulation study is to compare traditional sampling strategies with some MB procedures and some restricted representative sampling strategies in terms of estimation bias, efficiency, and confidence limit coverage probabilities for a variety of forestry populations. Reliability of the variance estimates is assessed.

Review of Literature

Linear models of the form

$$y_i = \alpha + \beta x_i + e_i \qquad [1]$$

where

$$E(e_i|x_i) = 0,\ V(e_i|x_i) = \sigma^2 x_i^k \text{ for } 0 \leq k \leq 2$$
$$E(e_i e_j|x_i x_j) = 0 \text{ for } i \neq j$$

are recognized as being appropriate in many forest mensurational applications (Spurr 1952) where y is the variable of interest (say, volume), x is an independent variable (say, D^2H) that is easier to measure than y but usually only of interest because of interest in y, and e denotes a random error. Relationships between other variables are often considered to be either curvilinear or nonlinear (say, tree height vs. tree diameter or tree height over age of tree) although such a nonlinear relationship may not exist or may not be obvious because of the scatter of the data (Schreuder et al. 1987).

Sample Selection

When there is a strong linear relationship between variables y and x, model-dependent (MD) procedures may be preferred over traditional probabilistic procedures. In MD procedures, inference is dependent on an assumed underlying model and samples may be drawn in a purposive (nonprobabilistic) manner to improve the efficiency of estimates.

There is no easily computed optimal sample for any $k > 0$ and $\alpha \neq 0$ in model [1]. The optimal sample is a function of the distribution of the x-values and hence depends on the population under consideration. Schreuder and Thomas (1985), Schreuder and Wood (1986), and Wood and Schreuder (1986 and references therein) compared MD and probabilistic procedures in simulation and actual surveys, primarily for timber volume estimation. They generally concluded that MD procedures can be more efficient than design-dependent (DD) procedures, but a big part of the improved efficiency is due to the use of a weighted regression estimator with the MD rather than the Horvitz-Thompson ratio estimator used with DD procedures. And, although the MD procedures with the regression estimator can have relatively small estimation bias and high efficiency for the populations studied, it is not clear why this should generally be true for other populations. For example, the pscx procedure was quite efficient for three timber sale and timber plot volume populations considered in Schreuder (1986) with the Horvitz-Thompson estimator. This was due to the elimination from the samples of the smallest and largest x-values (and hence the extreme y/x ratios) for the ratio estimator.

Karmel and Jain (1987) compared purposive and random sampling procedures to estimate capital expenditure. They found that a stratified sample consisting of the units with the largest x-values in each stratum with a simple ratio estimator was by far the most efficient sampling strategy. And they managed to obtain a meaningful estimate of error for this strategy.

Herson (1976) noted that if

$$y_k = \beta_1 x_k + \epsilon_k (x_k)^{1/2},\ k = 1,2,\ldots,N$$

where $\epsilon_1, \epsilon_2, \ldots, \epsilon_N$ are independent random variables with mean 0, variance σ^2, then the ratio-of-means estimator for the population total

$$\hat{T} = \left(\sum_{k=1}^{n} y_k / \sum_{k=1}^{n} x_k\right) X_T$$

is the best linear unbiased (BLU) predictor of Y_T with

$$MSE(\hat{T}) = E(\hat{T} - Y_T)^2 = \sigma^2[(X_T - n\bar{x})/n\bar{x}]X_T.$$

Here X_T denotes the population total for the x-values.

This estimator ($\hat{T}$) is also the BLU predictor for Y_T for the model

$$y_k = \beta_0 + \beta_1 x_k + \beta_2 x_k^2 + \ldots + \beta_J x_k^J + \epsilon_k (x_k)^{1/2}$$

when a balanced sample is selected. A balanced sample s(J) is one for which

$$\bar{x}^{(j)} = \bar{X}^{(j)}, \; j = 1,2,\ldots,J$$

where

$$\bar{x}^{(j)} = \frac{1}{n} \sum_{k=1}^{n} x_k^j \text{ and } \bar{X}^{(j)} = \frac{1}{N} \sum_{k=1}^{N} x_k^j .$$

Royall and Herson (1973a) noted that random sampling is fundamentally justified in that it provides samples that are approximately balanced (the sample means and variances are about the same as the corresponding population mean and variance for the covariate) with respect to various characteristics. Experimenters often reject random samples that appear to be "extreme" or "unrepresentative." After the sample is chosen and the data are gathered, adjustments are frequently made in calculations if the randomization process fails to divide the units into "sample" and "nonsample" groups that are similar with respect to relevant characteristics. One familiar technique of this type is post-stratification. This is also the basic rationale behind the ratio-of-means estimator that adjusts the sample mean $\bar{y}$ by the ratio adjustment $\bar{X}/\bar{x}$.

If not enough is known about the relationship between y and x to correct for imbalance in the sample, then a sample should be balanced on average, with respect to x such as in random sampling.

Iachan (1985) noted that the ratio-of-means estimator $\hat{T}$ can be badly biased. For example, for the model-based expectation (denoted by E_ξ)

$$E_\xi (y_i) = \beta_0 + \beta_1 x_i \text{ for } \beta_0 \neq 0$$

and

$$\text{Cov.}(y_i, y_j) = \sigma^2 x_i \text{ if } i = j$$
$$= 0 \text{ if } i \neq j,$$

the model bias of $\hat{T}$ becomes

$$E_\xi (\hat{T} - Y) = N \beta_0 (\bar{X} - \bar{x})/\bar{x},$$

which is small only if $|(\bar{x} - \bar{X})/|\bar{x}|$ is small. This leads naturally to the idea of balanced sampling such that $\bar{x} \doteq \bar{X}$.

Royall and Herson (1973b) considered stratification on a size variable as an additional technique to balance and protect against model failure. They showed that stratification and balanced sampling provide more protection for efficient estimation than balanced sampling alone using ratio-of-means estimators. If the population total is

$$T = \sum_{h=1}^{H} \sum_{k=1}^{N_h} y_{hk} = \sum_{h=1}^{H} T_h$$

where strata are defined so that stratum 1 contains the N_1 smallest units, stratum 2 the next N_2 smallest units, etc., for h strata and a sample of n_h units is selected from stratum h and the total T_h for the stratum is estimated by the ratio-of-means estimator

$$\hat{Y}_{h,rm} = (\sum_{s_h} y_{hk} / \sum_{s_h} x_{hk}) \sum_{k=1}^{N_h} x_{hk},$$

where s_h is the sample in stratum h, then the overall total T is estimated by the separate ratio-of-means estimator

$$\hat{T}^* = \sum_{h=1}^{H} \hat{Y}_{h,rm}.$$

This estimator can be generated by the stratified balanced sample of degree J denoted by s*(J) where $\bar{x}_h{}^j = \bar{X}_h{}^j$ for j = 1,2,...,J.

The main results of Royall and Herson (1973b) are given in the following two theorems:

Theorem 1: If n_h is proportional to $N_h \bar{X}_h{}^{1/2}$ then the strategy $[s^*(J), \hat{T}^*]$ is more efficient than

$$[s(J), \hat{Y}_{rm} = (\sum_{h=1}^{H} \sum_{k=1}^{n_h} y_{hk} / \sum_{h=1}^{H} \sum_{k=1}^{n_h} x_{hk}) X_T]$$

under model

$$y = \delta_0 \beta_0 + \delta_1 \beta_1 x + \delta_2 \beta_2 x^2 + \ldots + \delta_J \beta_J x^J + e$$

where $V(e) = \sigma^2 x$. That is,

$$E_\xi [\hat{Y}_{rm} - T]^2 \geq E_\xi (\hat{T}^* - T)^2.$$

Theorem 2: When the size measures $x_1,\ldots,x_N$ all have different values and the stratified balanced sampling strategy $[s^*(J), \hat{T}^*]$ with equal allocation is employed then the necessary conditions for optimal stratification under any and all models of the form

$$y = \delta_0 \beta_0 + \delta_1 \beta_1 x + \ldots + \delta_j \beta_j x^J + e$$

with $V(e) = \sigma^2 x$ are

(1) $N_1 \geq N_2 \geq \ldots \geq N_H$

and

(2) $N_1 \bar{X}_1 \leq N_2 \bar{X}_2 \leq \ldots \leq N_H \bar{X}_H$ if compatible with (1) above.

There are 3 special cases for which both (1) and (2) in Theorem 2 are satisfied:

(1) $N_1 = N_2 = \ldots = N_H$

(2) $N_1 \bar{X}_1 = N_2 \bar{X}_2 = \ldots = N_H \bar{X}_H$

(3) $N_1{}^2 \bar{X}_1 = N_2{}^2 \bar{X}_2 = \ldots = N_H{}^2 \bar{X}_H$.

Heuristic arguments, preliminary bits of empirical evidence, and theoretical calculations led Royall and Herson (1973b) to suggest that equal allocation in (3) above is usually close to the optimal allocation scheme.

Additional Estimators

Little (1983) discussed the merits of the generalized regression estimator

$$\hat{Y}_{GR} = \sum_{i=1}^{n} \Pi_i^{-1}(Y_i - \hat{u}_i) + \sum_{i=1}^{N} \hat{u}_i \qquad [2]$$

where $\hat{u}_i = \hat{\beta}x_i$ (or some other regression estimate) for the sampled and nonsampled units and Π_i is the probability of including unit i in the sample. For example if $x_i = 1$ for all i, then $\hat{u}_i$ can be a (possibly weighted) mean of the sampled values of y.

This estimator is asymptotically design unbiased. Särndal (1980) proposed calculating $\hat{\beta}$ by weighted least squares and discussed two possibilities:

(a) Π-inverse weighting in which each unit i is given a weight proportional to Π_i^{-1}. For this the estimator is called $\hat{Y}_{PI} = \hat{Y}_{GR}$.
(b) Best linear unbiased weighting, in which unit i is given a weight k_i^{-1} with k_i proportional to the residual variance of y_i under the regression model. For this the estimator is $\hat{Y}_{BLU} = \hat{Y}_{GR}$.

Estimator [2] can be viewed as a compromise between model-based and design-based inference. The second part of [2] is a prediction of $\hat{Y}$ from the regression model. The first term is the Horvitz-Thompson estimator applied to the residuals $Y_i - \hat{u}_i$ which protects the estimator from model misspecification by ensuring asymptotic design consistency. Särndal concluded that within this context, the choice of best linear unbiased weighting to calculate $\hat{\beta}$ seems more logical than Π-inverse weighting because the former yields an efficient estimate of β under the model. The best linear estimator is the maximum likelihood (ML) estimator under the normal linear regression model $Y_i \sim N(\beta^T x_i, k_i\sigma^2)$ where N(a, b) denotes the normal distribution with mean a, variance b, and independent Y_i's. The design weights yield ML estimates only if Π_i is proportional to k_i.

An alternative to [2] is the prediction estimator (PR)

$$\hat{Y}_{PR} = \{\sum_{i=1}^{n} Y_i + \sum_{i \notin s} \hat{\mu}_i\}, \qquad [3]$$

which is the sum of the observed and predicted values in the population.

Little (1983) argued that estimators of Y_T should be chosen from the class of estimators [3] rather than [2] because PR estimators have optimal statistical properties if the model is true, and from a frequentist perspective, PR estimators from least squares regression have minimum variance among unbiased linear estimators.

Variance Estimation

Royall and Cumberland (1978) discussed robust variance estimation for regression and ratio-of-means estimators under a linear regression model. For the ratio-of-means estimator

$$\hat{Y}_{RM} = N(\bar{y}/\bar{x})\bar{X}$$

the weighted least squares variance estimator is

$$v_1(\hat{Y}_{RM}) = v_L(\hat{Y}_{RM}) = (N/n)\frac{(N-n)}{(n-1)}\bar{X}\frac{\bar{x}_2}{\bar{x}}\sum_{i=1}^{n} r_i^2/x_i$$

where $\bar{x}_2$ is the mean of the x-values not sampled and $r_i = y_i - (\bar{y}/\bar{x})x_i$.

Robust variance estimators are

$$v_2(\hat{Y}_{RM}) = v_H(\hat{Y}_{RM}) = (N/n)(N-n)\frac{\bar{X}\,\bar{x}_2}{\bar{x}^2}\cdot$$

$$[(n-1)(1-c^2/n)]^{-1}\sum_{i=1}^{n} r_i^2$$

where

$$c^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i - \bar{x})^2/\bar{x}^2,$$

$$v_3(\hat{Y}_{RM}) = v_D(\hat{Y}_{RM}) = (N/n)(N-n)\frac{\bar{x}_2\,\bar{X}}{n\bar{x}^2}$$

$$\sum_{i=1}^{n} r_i^2[1 - \frac{x_i}{n\bar{x}}]^{-1},$$

and the standard variance estimator (Cochran 1977) is

$$v_4(\hat{Y}_{RM}) = v_c(\hat{Y}_{RM}) = [(N/n)(N-n)/(n-1)]\sum_{i=1}^{n} r_i^2.$$

For the simple linear regression estimator

$$\hat{Y}_{\ell r} = N[\bar{y} + \hat{\beta}(\bar{X} - \bar{x})]$$

where

$$\hat{\beta} = \sum_{i=1}^{n} y_i(x_i - \bar{x})/\sum_{i=1}^{n}(x_i - \bar{x})^2,$$

the weighted least squares variance estimator is

$$v_1(\hat{Y}_{\ell r}) = v_L(\hat{Y}_{\ell r}) = v_4(\hat{Y}_{\ell r})\{1 + (1-f)\,\Lambda_x^2/d\}$$

and robust variance estimators are

$$v_2(\hat{Y}_{\ell r}) = v_H(\hat{Y}_{\ell r}) = (N-n)s^2 + \frac{\alpha}{n^2d^2}(N-n)^2$$

$$\sum_{i=1}^{n} r_i^2[d + (x_i - \bar{x})\Lambda_x]^2$$

$$v_3(\hat{Y}_{\ell r}) = v_D(\hat{Y}_{\ell r}) = \frac{(N-n)^2}{d^2n(n-1)}$$

$$\sum_{i=1}^{n} r_i^2\{[d + (x_i - \bar{x})\Lambda_x]^2 + \frac{f}{1-f}d^2\}.$$

$$\{1 - \frac{(x_i-\bar{x})^2}{d(n-1)}\}^{-1}$$

and the standard variance estimator (Cochran 1977) is

$$v_4(\hat{Y}_{\ell r}) = v_c(\hat{Y}_{\ell r}) = \frac{N(N-n)}{n}s^2$$

where

$$s^2 = \sum_{i=1}^{n} r_i^2/(n-2)$$

$$r_i = y_i - \bar{y} - b(x_i - \bar{x})$$

$$\alpha^{-1} = 1-\{\sum_{i=1}^{n}[d + (x_i-\bar{x})\Lambda_x]^2\}^{-1}\sum_{i=1}^{n}[d+(x_i-\bar{x})\Lambda_x]^2 k_i$$

$$k_i = [1 + (x_i - \bar{x})^2/d]/n$$

$$\Lambda_x = \bar{x}_2 - \bar{x}$$

$$d = \sum_{i=1}^{n}(x_i - \bar{x})^2/n.$$

Royall and Cumberland's (1978) results, based on large sample theory, suggest that the variance estimators v_2 and v_3 may be more robust than the currently used v_1 and v_4. The weighted least squares variance estimators may not be robust enough in many applications. v_2 and v_3 are very similar to the jackknife variance estimator.

Royall and Cumberland (1981b) showed that under random sampling the bias-robust estimators $v_2(\hat{Y}_{\ell r})$, $v_3(\hat{Y}_{\ell r})$, and the jackknife variance estimator are clearly superior to $v_1(\hat{Y}_{\ell r})$ and $v_4(\hat{Y}_{\ell r})$ in estimating the actual mean square error as the sample mean $\bar{x}$ varies. For restricted random sampling, balanced so that $\bar{x} \doteq \bar{X}$, they found that

$$v_4(\hat{Y}_{\ell r}) \doteq v_2(\hat{Y}_{\ell r}) \doteq v_1(\hat{Y}_{\ell r}) < v_3(\hat{Y}_{\ell r}) < \bar{v}_J(\hat{Y}_{\ell r})$$

where the last one is the average jackknife variance estimator. For one population $\bar{v}_J$ was close to the actual mean square error. v_J appeared to be one of the best estimators for the other populations too.

Royall and Cumberland (1981a) showed that the estimators $v_2(\hat{Y}_{RM})$, $v_3(\hat{Y}_{RM})$, and the jackknife variance estimator are clearly superior to the weighted least squares estimator $v_1(\hat{Y}_{RM})$ for six study populations. They mentioned that the standard variance estimator $v_4(\hat{Y}_{RM})$ can be badly biased and proposed that

$$v_2(\hat{Y}_{RM}) = v_H(\hat{Y}_{RM}) = v_4(\hat{Y}_{RM})(\bar{x}_2\ \bar{X}/\bar{x}^2)(1 - v_s^2/n)^{-1}$$

where

$$v_s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/\{\bar{x}^2(n-1)\}$$

be used to adjust for this bias in $v_4(\hat{Y}_{RM})$. This bias problem with $v_4(\hat{Y}_{RM})$ may be the reason why it is not mentioned further in the results.

Schreuder and Anderson (1984) compared variance estimators for estimators of total volume for 2 populations of loblolly pine trees using $\hat{Y}_{\ell r}$, the simple linear (k = 0) regression estimator, and $\hat{Y}_{wr}$, the weighted linear (k = 1.5) regression estimator. A linear model with k = 1.5 is known to be very reasonable for these populations. They used estimators $v_1(\hat{Y}_{\ell r})$, $v_2(\hat{Y}_{\ell r})$, $v_3(\hat{Y}_{\ell r})$, $v_4(\hat{Y}_{\ell r})$, $v_J(\hat{Y}_{\ell r})$, $v_J(\hat{Y}_{wr})$, and $v_w(\hat{Y}_{wr})$ where the latter is the weighted regression estimator with k = 1.5. Samples of size 20, 40, 60, and 80 were used in the simulation study. They recommended the use of $v_J(\hat{Y}_{wr})$ for $\hat{Y}_{wr}$. The jackknife variance estimator $v_J(\hat{Y}_{\ell r})$ was less appropriate for the less appropriate estimator $\hat{Y}_{\ell r}$. For $\hat{Y}_{wr}$, $v_2(\hat{Y}_{wr})$ was less reliable and $v_3(\hat{Y}_{wr})$ was totally unreliable.

Efron (1982) discussed an alternative to jackknifing, called bootstrapping. This method is simpler to implement and requires fewer assumptions although it requires more computation. In bootstrapping, n_B samples of size n are randomly selected with replacement and the same estimator is used as with the sampling strategy in question. The n_B estimates are then treated as independent sample estimates and their variance yields an estimate of the variance of their average value (the bootstrap estimate) as well as for the original estimate.

Objectives

1. To compare simple random sampling (SRS), restricted (balanced) simple random sampling (RSRS), sampling with probability proportional to size (pps), stratified sampling with probability proportional to size (spps), purposive sampling from the cumulated x-values (pscx), and stratified sampling from the cumulated x-values (spscx) sampling designs with several ratio and regression estimators in terms of bias, variance, and mean square error for a wide range of populations from actual forestry data sets. These contain sets with known linear y-x relationships as well as others where the actual y-x relationship is more nebulous. Other criteria to be considered are reliability of variance estimator and coverage rates.

2. To determine if at least one reliable variance estimator for each sampling strategy (design and estimator) can be found.

Criteria for Evaluation of Sampling Strategies

a. Average estimator bias in percent of the true total,

$$\text{Percent Bias} = [(\hat{Y} - Y)/Y]\ {*}100\%.$$

b. Average iterated variance V_I for each sampling strategy.

c. Mean square error (MSE) = V_I + bias^2 for each sampling strategy, where bias = $\hat{Y} - Y$, and $V_I = \sum_{k=1}^{10000}(\hat{Y}_k - \hat{Y})^2/(10000-1)$, $\hat{Y}_k$ is the estimate at the k-th iteration.

d. Reliability of variance estimators expressed as a percentage of the best measure of the true variance, V_I;

$$\text{i.e., } [(\hat{V} - V_I)/V_I]\ {*}100\%,$$

where $\hat{V}$ is the average of variance estimates from 10,000 iterations.

e. Coverage probability of 95% confidence intervals; i.e., for targeted 95% confidence intervals, what percentage actually contains the parameter of interest?

Methods

Sampling Designs to be Tested

1. **SRS.**—All units have equal probabilities of selection and each pair of units has equal joint probabilities of selection.
2. **RSRS.**—Units are selected that are balanced on both the mean and variance of x. This is the same as SRS, except that samples with mean and variance of x more than a certain percentage away from the population mean and variance of x are rejected. Iachan (1985) pointed out that there will be sample units with zero probabilities of selection, and he gave modifications to avoid this. These, however, are cumbersome to implement, so they were not considered here.
3. **PPS.**—Units are selected with probability proportional to size x by sorting the population by x and sampling without replacement from the cumulated x-values.
4. **SPPS.**—Units are selected such that there are $H = n/2$ strata with
$$N_1^2\bar{X}_1 \doteq N_2^2\bar{X}_2 \doteq \dots \doteq N_H^2\bar{X}_H.$$
The strata are set up such that
$$\text{Max } \{|N_i^2 \bar{X}_i - N_j^2 \bar{X}_j|\}$$
is minimized for all i and j. An algorithm was written to set up the strata accordingly. Two units are selected within each of the H strata by using pps sampling within each stratum.
5. **PSCX.**—Divide the population sorted by x-values into n classes $[iX_T/n, (i + 1)X_T/n]$, $i = 0,1,\dots,n - 1$ and calculate the number of units N_i in each class. Within each class select the first unit that is within $p = 5\%$ of X_T/nN_i (if no unit satisfies this criterion, p is boosted to 10%, 15%, etc., until a unit is selected). Here X_T is the sum of the x-values in the population and the units are randomly visited.
6. **SPSCX.**—Randomly select 1 unit within each of the n classes set up for pscx sampling. This is stratified sampling, selecting 1 unit per stratum.

Estimators to be Tested

1. $$\hat{Y}_{HT} = \sum_{i=1}^{n} (y_i/nx_i)\, X_T \qquad [4]$$

or for the stratified pps procedures (spps)

$$\hat{Y}_{HT} = \sum_{h=1}^{H} \sum_{i=1}^{2} (y_{hi}/2x_{hi})X_{ht},$$

where X_{ht} = total for x in stratum h.

Variance estimators are

a. $$v_1 (\hat{Y}_{HT}) = (N\bar{X}^2/f) \sum_{i=1}^{n} (d_i/x_i)^2/(n-1) \qquad [5]$$

where $f = n/N$, $d_i = y_i - bx_i$, and $b = \sum_{i=1}^{n} (y_i/nx_i)$.

This is the standard pps with replacement variance estimator which should be satisfactory for without replacement sampling for the large forestry populations usually of interest. For the stratified pps procedures, the standard pps with replacement variance estimator

$$v_2 (\hat{Y}_{HT}) = \sum_{h=1}^{H} [\sum_{i=1}^{2} (y_i/x_i)^2 X_{ht}^2 - 2\, \hat{Y}_{HT}{}^2]/2 \qquad [6]$$

was used.

b. $$v_J(\hat{Y}_{HT}) = (1 - f)[N\bar{X}^2/f] \sum_{i=1}^{n} (d_i/x_i)^2/(n - 1) \qquad [7]$$

v_J is also defined in equation [19].

This is the jackknife variance estimator v_J, which is the pps with replacement variance estimator $v_1 (\hat{Y}_{HT})$ multiplied by the correction term $(1 - f)$. It is recommended for $\hat{Y}_{HT}$ by Cumberland and Royall (1981). This may perform better than $v_1(\hat{Y}_{HT})$ for small populations.

c. $v_B(\hat{Y}_{HT})$ is shown in equation [21].

2. $$\hat{Y}_{RM} = [\sum_{i=1}^{n} y_i / \sum_{i=1}^{n} x_i]X_T \qquad [8]$$

a. $$v_H (\hat{Y}_{RM}) = (N/n)\,(N - n)\, \frac{\bar{X}\bar{x}_2}{\bar{x}^2} [(n - 1)\,(1 - c^2/n)]^{-1} \sum_{i=1}^{n} r_i^2, \qquad [9]$$

where $c^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i-\bar{x})^2/\bar{x}^2$, $r_i = Y_i - (\bar{y}/\bar{x})x_i$,

and $\bar{x}_2$ is the sample mean of the nonsampled units.

b. $v_J(\hat{Y}_{RM})$ is defined in equation [19].

c. $$v_B(\hat{Y}_{RM}) = \frac{\sum_{i=1}^{n_B} (Y_{Bi} - \bar{Y}_B)^2}{(n_B - 1)}$$

or as defined in equation [21].

3. $$\hat{Y}_{LR} = N\,[\hat{\alpha} + \hat{\beta}\, \bar{X}] \qquad [10]$$

with variance estimators

a. $$v(\hat{Y}_{LR}) = \frac{N^2(1-\frac{n}{N})}{n(n-2)} [\Sigma(y_i-\bar{y})^2 - \frac{\{\Sigma(y_i-\bar{y})\,(x_i-\bar{x})\}^2}{\Sigma(x_i-\bar{x})^2}] \qquad [11]$$

which is the standard that is used for comparison purposes (Cochran 1977).

b. $v_H(\hat{Y}_{LR}) = (N - n)s^2 + \frac{\alpha}{n^2d^2}(N - n)^2$

$$\sum_{i=1}^{n} r_i^2[d + (x_i - \bar{x})\Lambda_x]^2 \qquad [12]$$

where $d = \sum_{i=1}^{n}(x_i - \bar{x})^2/n$,

$\Lambda_x = \bar{x}_2 - \bar{x}$,

$\alpha^{-1} = 1 - \{\sum_{i=1}^{n}[d + (x_i - \bar{x})\Lambda_x]^2\}^{-1}$

$$\sum_{i=1}^{n}[d + (x_i - \bar{x})\Lambda_x]^2 k_i$$

where $k_i = [1 + (x_i - \bar{x})^2/d]/n$.

c. $v_J(\hat{Y}_{\ell r})$ is defined in equation [19].

d. $v_B(\hat{Y}_{\ell r})$ is defined in equation [21].

4. $$\hat{Y}_{BLU} = \left[\frac{\sum_{i=1}^{n} y_i/x_i^{1.5}}{\sum_{i=1}^{n} 1/x_i^{1.5}} + \hat{\beta}_{BLU}\left(\bar{X} - \frac{\sum_{i=1}^{n} x_i/x_i^{1.5}}{\sum_{i=1}^{n} 1/x_i^{1.5}}\right) + \sum_{i=1}^{n} D_i/N\Pi_i\right]*N \qquad [13]$$

where

$$\hat{\beta}_{BLU} = \frac{\sum_{i=1}^{n} 1/x_i^{1.5}\sum_{i=1}^{n} x_iy_i/x_i^{1.5} - \sum_{i=1}^{n} y_i/x_i^{1.5}\sum_{i=1}^{n} x_i/x_i^{1.5}}{\sum_{i=1}^{n} 1/x_i^{1.5}\sum_{i=1}^{n} x_i^2/x_i^{1.5} - (\sum_{i=1}^{n} x_i/x_i^{1.5})^2}$$

and

$$D_i = y_i - \frac{\sum_{i=1}^{n} y_i/x_i^{1.5}}{\sum_{i=1}^{n} 1/x_i^{1.5}} - \hat{\beta}_{BLU}\left(x_i - \frac{\sum_{i=1}^{n} x_i/x_i^{1.5}}{\sum_{i=1}^{n} 1/x_i^{1.5}}\right)$$

(Särndal 1980)

with

a. $v_J(\hat{Y}_{BLU})$ as given in equation [19]

b. $v_B(\hat{Y}_{BLU})$ as given in equation [21]

c. $v_W(\hat{Y}_{BLU})$ as given in equation [17].

5. $$\hat{Y}_{PR} = [\sum_{i=1}^{n} Y_i + \sum_{i \notin s}\hat{\mu}_i] = n\bar{y} + \sum_{i \notin s}\hat{\mu}_i \qquad [14]$$

where $\hat{\mu}_i = \hat{\alpha} + \hat{\beta}x_i$

where $\hat{\beta} = \hat{\beta}_{BLU}$

and $\hat{\alpha} = \hat{\alpha}_{BLU} = \bar{y} - \hat{\beta}_{BLU}\,\bar{x}$

with

a. $v_J(\hat{Y}_{PR})$ as given in equation [19]

b. $v_B(\hat{Y}_{PR})$ as given in equation [21]

c. $v_W(\hat{Y}_{PR})$ as given in equation [17]. We will use $k = 1.5$ at this time.

6. $$\hat{Y}_{WR} = N[\hat{\alpha}_w + \hat{\beta}_w\bar{X}] \qquad [15]$$

with

$$\hat{\alpha}_W = \{\sum_{i=1}^{n} y_ix_i^{-k} - b_w\Sigma x_i^{1-k}\}/\sum_{1=1}^{n} x_i^{-k}$$

and

$$\hat{\beta}_W = \frac{\sum_{i=1}^{n} 1/x_i^k\sum_{i=1}^{n} y_ix_i^{1-k} - \sum_{i=1}^{n} x_i^{1-k}\sum_{i=1}^{n} y_ix_i^{-k}}{\sum_{i=1}^{n} 1/x_i^k\sum_{i=1}^{n} x_i^{2-k} - (\sum_{i=1}^{n} x_i^{1-k})^2}$$

$\hat{Y}_{WR}$ and $\hat{Y}_{BLU}$ are the same except that $\hat{Y}_{BLU}$ contains the error correction $\sum_{i=1}^{n} D_i/n\Pi_i$ with variance estimators

a. $v(\hat{Y}_{WR})$ as defined in equation [17] with $\Pi_i = x_i^k$

b. $v_J(\hat{Y}_{WR})$ as defined in equation [19]

c. $v_B(\hat{Y}_{WR})$ as defined in equation [21].

7. $$\hat{Y}_{PI} = \left[\frac{\sum_{i=1}^{n} y_i/\Pi_i}{\sum_{i=1}^{n} 1/\Pi_i} + \hat{\beta}_{PI}\left(\bar{X} - \frac{\sum_{i=1}^{n} x_i/\Pi_i}{\sum_{i=1}^{n} 1/\Pi_i}\right)\right] * N \qquad [16]$$

(Särndal 1980)

with

$$\hat{\beta}_{PI} = \frac{\sum_{i=1}^{n} 1/\Pi_i\sum_{i=1}^{n} x_iy_i/\Pi_i - \sum_{i=1}^{n} y_i/\Pi_i\sum_{i=1}^{n} x_i/\Pi_i}{\sum_{i=1}^{n} 1/\Pi_i\sum_{i=1}^{n} x_i^2/\Pi_i - (\sum_{i=1}^{n} x_i/\Pi_i)^2}.$$

Here Π_i is the probability of selecting unit i in the sample, which is taken to be

$\Pi_i = n/N$ for SRS and RSRS

$= 1/N_i$ for spscs where N_i = the number of units in the n strata set up

$= nx_i/X_t$ for pps and spps, where $X_t = X_T$ for pps, $X_t = X_{ht}$ for spps ($h = 1, \ldots, H$).

Clearly the probabilities used are only approximations for RSRS.

a. with $v_W(\hat{Y}_{PI})$ as defined in equation [17]

b. with $v_J(\hat{Y}_{PI})$ as defined in equation [19]

c. with $v_B(\hat{Y}_{PI})$ as defined in equation [21].

The variance formulas for $\hat{Y}_{PI}$, $\hat{Y}_{BLU}$, and $\hat{Y}_{PR}$ are quite messy. Särndal (1982) proposes a variance estimator for generalized regression estimators of linear functions (Schreuder et al. 1987):

$$v(\hat{Y}_{GR}) = s^2[\sum_{i=1}^{N}\Pi_i - \sum_{i=1}^{n}\Pi_i + \{[(\sum_{i=1}^{n} x_i^2/\Pi_i)(N - n)$$

$$-2(X_T - n\,\bar{x})\sum_{i=1}^{n}(x_i/\Pi_i)]\,(N-n)/\sum_{i=1}^{n}1/\Pi_i$$

$$+(X_T - n\bar{x})^2]/SS_{xw}\} \qquad [17]$$

where GR = PI or BLU or PR and

$$s^2 = [\sum_{i=1}^{n} y_i^2/\Pi_i - (\sum_{i=1}^{n} y_i/\Pi_i)^2/\sum_{i=1}^{n} 1/\Pi_i - \hat{\beta}^2 SS_{xw}]/(n-2)$$

with $\hat{\beta}$ being either $\hat{\beta}_{PI}$, $\hat{\beta}_{BLU}$, or $\hat{\beta}_{PR}$ and

$$SS_{xw} = \sum_{i=1}^{n} x_i^2/\Pi_i - (\sum_{i=1}^{n} x_i/\Pi_i)^2/\sum_{i=1}^{n} 1/\Pi_i$$

with Π_i = the probability of selecting unit i in a sample of n units for $v(\hat{Y}_{PI})$ and $\Pi_i = X_i^k$, where we use k = 1.5 in this study for $v(\hat{Y}_{BLU})$ and $v(\hat{Y}_{PR})$.

Proposed General Robust Variance Estimators

Both the jackknife and the bootstrap variance estimators seem to be promising robust variance estimators for complex sampling strategies (Efron 1982) and hence were tested in this simulation study.

The jackknife estimator, generally defined, is

$$\hat{Y}_J = \sum_{i=1}^{n} \hat{Y}_{Ji}/n \qquad [18]$$

where n is the sample size and

$$\hat{Y}_{Ji} = n\,\hat{Y} - (n-1)\hat{Y}_{(i)}$$

where $\hat{Y}$ is the standard estimator used and $\hat{Y}_{(i)}$ is the standard estimator with the i-th unit deleted. Then

$$v(\hat{Y}_J) = \frac{(N-n)}{N}\sum_{i=1}^{n}(\hat{Y}_{Ji} - \hat{Y}_J)^2/n(n-1). \qquad [19]$$

The bootstrap estimator is

$$\hat{Y}_B = \sum_{i=1}^{n_B} \hat{Y}_{Bi}/n_B, \qquad [20]$$

where n_B = number of bootstrap samples selected, and $\hat{Y}_{Bi}$ is the i-th bootstrap estimate. This estimator has variance

$$v(\hat{Y}_B) = \sum_{i=1}^{n_B}(\hat{Y}_{Bi} - \hat{Y}_B)^2/(n_B-1). \qquad [21]$$

Weighted jackknife and bootstrap variance estimators were not considered because the theory was not sufficiently advanced at the time.

Simulation Statistics

For each of the 6 sample designs 10,000 samples of 20 units were selected. For each sample the 7 estimates and the variance estimators were computed. This was done for each population. Bootstrapping with n_B = 50 was done for each sample as described below. Bootstrapping takes considerable computing time so only 2,000 samples are selected for bootstrapping. Jackknifing was done by simply deleting each of the 20 sample units one at a time, generating 20 pseudo-estimates for each of the 7 estimators, and computing the variance between them using equation [19].

In bootstrapping for regression the following approach was used for each sample selection method:

1. The regression coefficients $\hat{\alpha}$ and $\hat{\beta}$ were computed. In the case of $\hat{Y}_{PI}$,

$$\hat{\alpha}_{PI} = \left[(\sum_{i=1}^{n} y_i/\Pi_i)/(\sum_{i=1}^{n} 1/(\Pi_i) - \hat{\beta}_{PI}(\sum_{i=1}^{n} x_i/\Pi_i)/\sum_{i=1}^{n} 1/\Pi_i \right].$$

2. $e_i = y_i - \hat{\alpha}_{PI} - \hat{\beta}_{PI}\,x_i$ for all n values were computed.
3. n values were drawn with replacement from the e_i-values and added to $\hat{y}_i = \hat{\alpha}_{PI} + \hat{\beta}_{PI}\,x_i$ so that n values $y_i^* = \hat{y}_i + e_i^*$ were obtained.
4. $\hat{\alpha}_{PI}^B$ and $\hat{\beta}_{PI}^B$ for the n values of y_i^*, x_i were computed.
5. $\hat{Y}_{PI}(B) = N\,(\hat{\alpha}_{PI}^B + \hat{\beta}_{PI}^B\,\bar{X})$ was computed.

Then the variance between the n_B estimates Y_{PI} (B) yielded the bootstrap variance for $\hat{Y}_{PI}$. The bootstrapping technique described above is for simple random sampling. For other sampling designs the probability of selection for bootstrapping should mimic the probability of selection for the original sample.

Similarly for $\hat{Y}_{BLU}$

$$\hat{\alpha}_{BLU} = (\sum_{i=1}^{n} y_i/x_i^{1.5})/\sum_{i=1}^{n} 1/x_i^{1.5}$$

$$+\hat{\beta}_{BLU}\{\sum_{i=1}^{n} D_i/(N\,\Pi_i) - (\sum_{i=1}^{n} x_i/x_i^{1.5})/(\sum_{i=1}^{n} 1/x_i^{1.5})\}$$

and for $\hat{Y}_{PR}$ we used

$$\hat{\alpha}_{PR} = \hat{\alpha}_{BLU}$$

$$\hat{\beta}_{PR} = \hat{\beta}_{BLU}\,.$$

For each estimator we computed both the average bootstrap estimate (i.e., $\hat{Y}_{BLU}$ (B)) and its average variance estimate.

For SRS, RSRS, pscx, and spscx, we randomly selected n units out of n units with replacement and computed for the b-th sample (b = 1, ..., n_B):

$$\hat{Y}_{HT}{}^b = (\sum_{i=1}^{n} y_i^*/n\,x_i^*)X_T$$

$$\hat{Y}_{RM}{}^b = (\sum_{i=1}^{n} y_i^*/\sum_{i=1}^{n} x_i^*)X_T$$

and then computed the variability between the n_B estimates using equation [21]. For each estimator we were interested in both the average bootstrap estimate (i.e., $\hat{Y}_{HT}$ (B)) and its average variance estimate.

For pps we did the following:

1. Drew n units with pps with replacement from the n sample units (cumulated the x-values in the original sample; selected n random numbers between 1 and the sum of the x's and selected the units corresponding to the selected random numbers).

2. Computed $\hat{Y}_{HT}{}^{b} = \sum_{i=1}^{n} (y_i^*/n\ x_i^*)X_T$

   and

$$\hat{Y}_{RM}{}^{b} = (\sum_{i=1}^{n} y_i^* / \sum_{i=1}^{n} x_i^*)X_T$$

   and computed the variability between estimates using equation [21]. For spps, we drew 2 units by pps with replacement from the 2 units in each stratum and computed

$$\hat{Y}_{HT}{}^{b} = (\sum_{i=1}^{n} y_i^*/n\ x_i^*)X_T$$

   and

$$\hat{Y}_{RM}{}^{b} = (\sum_{i=1}^{n} y_i^* / \sum_{i=1}^{n} x_i^*)X_T\ .$$

Rao and Wu (1988) point out that such nonlinear estimators as the ratio estimators are not consistent estimators of the true variances of $\hat{Y}_{HT}$ (and $\hat{Y}_{RM}$). They suggest as alternatives for $\hat{Y}_{HT}$ the following estimators that do yield consistent variance estimators of the true variances of $\hat{Y}_{HT}$. To do this we need estimated totals for each stratum for the original sample; i.e., for stratum h:

$$\hat{Y}_{HT}(h) = \sum_{i=1}^{2} (y_{hi}/2\ x_{hi})X_{ht}$$

where X_{ht} is the total for stratum h.

Then for the with-replacement sample they compute

$$\hat{Y}_{HT(RW)}^{(b)} = \sum_{h=1}^{H} \tilde{z}_h,\ b = 1,\ldots,B$$

where for the two units in each stratum

$$\tilde{z}_h = \hat{Y}_{HT}(h) + \sqrt{2}\{(\hat{Y}_{HT}{}^*(h) - \hat{Y}_{HT}(h)\}$$

and

$$\hat{Y}_{HT}(h) = \sum_{i=1}^{2} (y_{hi}/2\ x_{hi})X_{ht}$$

Then compute the bootstrap variances of $\hat{Y}_{HT}$ (RW) using equation [21]. We are interested in the average bootstrap variance estimates.

Populations

1. **Simulated loblolly pine population.**—This data set of 1,795 trees was generated from the loblolly pine volume data set (population 2) (6 trees with negative values for y were eliminated). The variables are total bole volume in cubic feet (v_c) and tree d.b.h. in inches squared times total height (D^2H) in feet. For each tree the value of v_c was computed from the following equation:

$$v_c = -0.734 + 0.002129\ D^2H + e$$

with residual error e generated from

$$N(0,\ \sigma^2(D^2H)^{1.5}),$$

where $\sigma^2 = 0.00001$. The regression coefficients were obtained by fitting weighted regression (k = 1.5) to the loblolly pine volume data (see population 2). The population skewness of D^2H is 2.552, and the kurtosis is 8.047. This data set was used to check how the estimates perform with normally distributed errors.

2. **Loblolly pine population.**—This data set consists of N = 1,801 *Pinus taeda* trees with measurement v_c = total bole volume (cubic feet), D = diameter breast height at 4 feet 5 inches (in inches) and total height (in feet). The data set was selected from the large data set discussed in McClure et al. (1983) so as to reflect the actual diameter distribution in the Coastal Plains of the southeastern United States (McClure et al. 1987). The weighted regression relationship for the data set that describes the actual relationship quite well is

$$v_c = -0.734 + 0.002129\ D^2H + e$$

with $v(e) = \sigma^2\ (D^2H)^k$, where e = residual error, k = 1.5 = a constant of heterogeneity, and $R^2 = 0.98$ for simple linear regression. The population skewness of D^2H is 2.556, and the kurtosis is 8.069. This data set was also used in Schreuder and Thomas (1985).

3. ***Pinus radiata* permanent yield plot data, Kowen Forest, Australia Capital Territory.**—The variables are merchantable volume (in m^3) and D^2H (in cm^2m). This population comprises 47 plots and 2,761 trees. Plots were randomly located within site index classes within compartments in 1955 and remeasured in 1958, 1962, and 1971. Some plots were measured at other times for special purposes. Plot size is 2 by 1 chain or 0.08 ha.

The diameter of each stem was measured at breast height (4 feet 3 inches) and four bark thickness measurements were taken at 5 feet and 15 feet, to derive underbark taper. Total height was measured by Haga altimeter.

Merchantable volume of the trees was derived from a 4-variable tree volume table for *Pinus radiata* (Lewis et al. 1973). Volume to a 10-inch diameter inside bark (V_{ib}) in 1955 and D^2H in 1955 were used as variables. The weighted regression (k = 1.5) was

$$V_{ib} = -0.02085 + 0.0000258\ D^2H$$

and the simple regression was

$$V_{ib} = -0.0177 + 0.0000254\ D^2H\ (R^2 = 0.9517).$$

The weighted regression describes the actual relationships of the variables quite well. The population skewness of D^2H is 2.088, and the kurtosis is 5.278.

4. **Southern undisturbed remeasurement plots.**—This data set consists of 275 remeasured undisturbed forest

plots in the southern United States. Net plot cubic foot volumes at times $t(v_t)$ and $t-5$ (v_{t-5}) are used. Plots were measured 5 years apart. The weighted regression ($k = 1.5$) is

$$v_t = 206.37 + 0.812\ V_{t-5}$$

and the simple linear regression is

$$v_t = 141.55 + 0.863\ v_{t-5}$$

($R^2 = 0.73$ for simple linear regression). This is a good description of the data although whether $k = 1.5$ cannot be verified reliably for such a small data set. The population skewness of v_{t-5} is 1.331, and the kurtosis is 5.482.

5. **Pine Creek periodic management inventory data, Australia (N = 2,748).**—Tree diameters (in cm) measured in 1979 (D79) and in 1967 (D67) are used as variables. From 1958 to 1961, 114 5- by 1-chain (0.5-acre or 0.202-ha) rectangular plots were located at random within a native eucalypt forest. Plots were established in each of 43 compartments depending on the size of the compartment. Two, three, or four plots were distributed within a compartment according to size of the compartment and the relative area of forest types so that the units were allocated roughly proportional to the area by types. Seventy-one plots were established in the "ridge type" (blackbutt and mixed hardwood types) and 43 in the "gully type" (the flooded gum type). One of the gully plots was destroyed in 1962.

The remaining plots were remeasured in 1963 and every 1–3 years thereafter up to 1981 or until destroyed or discarded. The variables and their weighted and simple linear regression are

D79 = 1.906 + 1.089 D67 for weighted regression and

D79 = 3.246 + 1.052 D67 for simple linear regression ($R^2 = 0.9463$). The population skewness of D67 is 2.085, and the kurtosis is 7.250.

6. **Alabama remeasurement data.**—The data set consists of 1,905 remeasured undisturbed volume plots in Alabama. Plot cubic foot volume measured in 1982 (v_{1982}) and 1973 (v_{1973}) were used. The weighted regression relationship is

$$v_{1982} = 639.78 + 0.829\ v_{1973} \qquad \text{for } k = 1.5.$$

There is a definite linear relationship between the variables but it is not clear that there is heterogeneity of variance. The simple linear regression is

$$v_{1982} = 632.39 + 0.856\ v_{1973}\ (k = 0)$$

($R^2 = 0.5831$ for simple linear regression).

The population skewness of v_{1975} is 1.327, and the kurtosis is 2.515. This data set was used in Schreuder and Thomas (1985) too.

7. ***Pinus radiata* bark data, Australian Capital Territory (A.C.T.), Australia.**—Relative bark thickness (RBT) (ratio of bark thickness at a point on the bole to the bark thickness at breast height) and relative diameter to the fourth power (RD^4) (ratio of diameter over bark at the same point on the bole to d.b.h. outside bark) were used as variables. The data comprise measurements made by diameter tape both outside and inside bark at 1- to 2-m intervals along the boles of 446 trees ranging in age from 18 to 45 years (number of measurements = 5,142).

The data were derived for the purpose of developing tree volume, taper, and bark thickness functions for radiata pine in the A.C.T. and derived from plantation forests. The weighted linear regression is

$$RBT = 0.1046 + 1.1060\ RD^4$$

and simple linear regression is

$$RBT = 0.1316 + 0.7904\ RD^4\ (R^2 = 0.8817).$$

The population skewness of RD^4 is 1.355, and the kurtosis is 1.130.

8. **New York (NY) remeasurement data.**—The data set consists of 622 remeasured volume plots in New York. It excludes observations with zero values. The variables are gross cubic foot volume per acre of live trees (GAL) at times t and t-1. The weighted regression is

$$GAL_t = 698.7566 + 0.8802059\ GAL_{t-1}.$$

The simple linear regression is

$$GAL_t = 1053.706 + 0.53\,13058\ GAL_{t-1}\ (R^2 = 0.2156).$$

The population skewness of GAL_{t-1} is 0.987, and the kurtosis is 1.832.

Results and Discussion

In a study of this type, the amount of results generated easily overwhelm one's ability to absorb what is going on. Hence we used a sequential approach. We evaluated estimator bias as a percentage of the true total, where less than 0.20% bias (<0.20%) is considered no bias, 0.20–1.00% is small bias, 1–5% is serious, and >5% is very serious bias. For variance estimation, bias of less than 5% (of the iterated variance) is considered no bias, 5–10% is small bias, 10–20% is moderate bias, 20–50% is serious bias, and >50% is very serious bias. For all populations, the results for 10,000 iterations are given, except that only 2,000 bootstrap samples could be run. There was little or no change in results from 2,000 to 10,000 iterations, indicating that all average estimates had become quite stable.

Estimation Bias (table 1)

If an unbiased estimator is wanted across all populations, clearly only pps and spps with YHT satisfy this requirement. Only YPI with spps, YLR (= YPI) with SRS and RSRS; and YBLU, YWR, and YPI with spscx have essentially no estimation bias across all eight populations.

If only the populations are considered where linearity is a reasonable assumption (populations 1–6), YBLU and YWR are essentially unbiased across all six populations for all methods except pscx. Both estimators show no bias for spps. For these six populations, all SRS and RSRS regression estimators show small or no bias as does YPI for spscx. For populations 7 and 8, where the assumption of linearity is less reasonable, no unbi-

Table 1.—Bias expressed as percent of the true total for the seven classical estimators (mean square errors in parentheses) with the six sample selection methods with n = 20 and 10,000 iterations from each population (2,000 iterations for bootstrap variance estimates).[1]

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[4]
1. Simulated loblolly pine, volume (MSE * 10^6), N = 1795							
SRS	−0.56		−0.14	−0.13	−0.13	−0.19	−0.14
	(1.24)		(1.26)	(1.13)	(1.13)	(1.07)	(1.26)
RSRS	−0.03		0.01	0.01	0.01	−0.09	0.01
	(1.06)		(1.06)	(1.05)	(1.05)	(0.99)	(1.06)
pps		0.02	−0.31	−0.04	0.10	−0.01	−0.05
		(0.80)	(2.11)	(0.80)	(0.89)	(0.79)	(1.81)
spps		−0.02	−0.13	−0.03	0.05	−0.02	−0.03
		(0.67)	(0.89)	(0.67)	(0.76)	(0.66)	(0.67)
spscx			−0.37	−0.05	0.11	−0.03	−0.05
			(1.98)	(0.69)	(0.79)	(0.66)	(0.68)
pscx			−1.49	0.07	0.07	−0.11	
			(2.00)	(0.73)	(0.73)	(0.64)	
2. Loblolly pine, volume (MSE * 10^6), N = 1801							
SRS[4]	−0.62		−0.49	−0.20	−0.20	−0.58	−0.49
	(1.31)		(1.01)	(1.12)	(1.12)	(0.77)	(1.01)
RSRS[4]	−0.90		−0.86	−0.87	−0.87	−0.96	−0.86
	(0.83)		(0.84)	(0.83)	(0.83)	(0.69)	(0.84)
pps		0.04	−6.16	−0.19	1.67	0.08	−0.34
		(0.58)	(6.33)	(0.53)	(1.36)	(0.58)	(0.64)
spps		−0.03	−1.46	−0.05	1.11	0.11	−0.05
		(0.56)	(0.70)	(0.56)	(1.40)	(0.60)	(0.56)
spscx			−6.29	−0.00	1.92	0.25	−0.01
			(4.88)	(0.43)	(1.16)	(0.46)	(0.43)
pscx			−4.44	0.56	0.56	−0.44	
			(2.28)	(0.62)	(0.62)	(0.42)	
3. *Pinus radiata*, volume (MSE * 10^2), N = 2761							
SRS	−0.42		0.20	−0.01	−0.01	0.06	0.20
	(7.30)		(7.00)	(6.47)	(6.47)	(5.90)	(7.00)
RSRS	0.34		0.42	0.41	0.41	0.42	0.42
	(6.11)		(6.15)	(6.05)	(6.05)	(5.61)	(6.15)
pps		−0.00	1.87	−0.05	−0.56	−0.05	0.12
		(5.34)	(10.84)	(4.67)	(5.41)	(4.70)	(4.83)
spps		−0.01	0.56	−0.01	−0.41	−0.06	−0.01
		(4.38)	(4.82)	(4.37)	(5.15)	(4.40)	(4.38)
spscx			2.06	−0.03	−0.75	−0.14	−0.03
			(9.33)	(4.15)	(4.91)	(4.13)	(4.14)
pscx			3.69	−1.69	−1.69	−0.54	
			(11.73)	(5.59)	(5.59)	(4.22)	
4. Southern, remeasured (MSE * 10^8), N = 275							
SRS	0.09		−0.26	−0.24	−0.24	−0.55	−0.26
	(8.19)		(8.28)	(8.36)	(8.36)	(8.56)	(8.28)
RSRS	−0.08		−0.16	−0.18	−0.18	−0.46	−0.16
	(8.16)		(8.17)	(8.14)	(8.14)	(8.12)	(8.17)
pps		0.19	−1.15	−0.08	0.33	0.03	−0.13
		(10.24)	(9.45)	(9.13)	(10.27)	(9.38)	(9.03)
spps		−0.05	−0.50	−0.14	−0.13	−0.09	−0.13
		(7.83)	(7.15)	(7.31)	(8.11)	(7.28)	(7.39)
spscx			−0.76	−0.09	0.25	−0.03	−0.10
			(8.28)	(8.50)	(10.10)	(8.75)	(8.52)
pscx			−0.57	−2.33	−2.33	−1.94	
			(7.33)	(7.28)	(7.28)	(6.91)	
5. Pine Creek, diameter (MSE * 10^6), N = 2748							
SRS	0.05		0.17	0.14	0.14	0.22	0.17
	(6.77)		(6.52)	(6.70)	(6.70)	(6.72)	(6.52)
RSRS	0.11		0.10	0.11	0.11	0.17	0.10
	(6.31)		(6.24)	(6.30)	(6.30)	(6.37)	(6.24)
pps		0.01	0.42	0.11	−0.06	0.07	0.11
		(6.93)	(7.36)	(7.13)	(7.31)	(7.14)	(7.10)
spps		0.03	0.16	0.05	−0.10	−0.02	−0.05
		(6.13)	(6.12)	(6.10)	(6.21)	(6.12)	(6.09)
spscx			0.33	−0.01	−0.19	−0.05	−0.01
			(6.80)	(6.60)	(6.71)	(6.59)	(6.59)
pscx			0.39	−0.09	−0.09	0.03	
			(6.85)	(6.69)	(6.69)	(6.62)	

Table 1.—Continued

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[4]
6. Alabama, remeasured (MSE * 10^{11}), N = 1905							
SRS	1.20		0.26	0.14	0.14	0.60	0.26
	(1.04)		(0.60)	(0.69)	(0.69)	(0.86)	(0.60)
RSRS	0.40		0.16	0.16	0.16	0.64	0.16
	(0.62)		(0.57)	(0.58)	(0.58)	(0.71)	(0.57)
pps		0.22	1.81	0.59	−0.02	0.44	0.57
		(1.88)	(0.84)	(0.89)	(1.08)	(0.92)	(0.87)
spps		0.05	0.77	0.13	−0.45	0.14	0.12
		(0.80)	(0.58)	(0.59)	(0.72)	(0.60)	(0.60)
spscx			1.47	0.14	−0.71	−0.09	0.12
			(0.75)	(0.82)	(1.16)	(0.88)	(0.82)
pscx			1.39	0.17	0.17	0.58	
			(0.78)	(1.30)	(1.30)	(1.00)	
7. *Pinus radiata*, bark (MSE * 10^4), N = 5142							
SRS	2.75		−0.58	1.24	1.24	9.52	−0.58
	(5.27)		(0.81)	(3.49)	(3.49)	(17.46)	(0.81)
RSRS	−0.09		−0.36	−0.19	−0.19	8.61	−0.36
	(0.87)		(0.71)	(0.79)	(0.79)	(11.29)	(0.71)
pps		0.02	−8.20	−0.04	1.90	0.27	−1.07
		(57.59)	(3.79)	(2.25)	(2.93)	(2.34)	(2.23)
spps		0.44	−1.95	1.44	2.39	1.58	0.77
		(38.36)	(0.94)	(0.90)	(1.64)	(0.90)	(0.86)
spscx			−7.89	0.22	2.05	0.30	0.20
			(2.97)	(1.85)	(2.91)	(1.92)	(1.85)
pscx			−7.14	6.52	6.52	3.57	
			(2.69)	(4.46)	(4.46)	(2.59)	
8. New York, remeasured (MSE * 10^{11}), N = 622							
SRS	1.63		0.95	0.79	0.79	3.50	0.95
	(0.26)		(0.14)	(0.17)	(0.17)	(0.23)	(0.14)
RSRS	1.23		0.96	1.01	1.01	3.47	0.96
	(0.14)		(0.13)	(0.14)	(0.14)	(0.19)	(0.13)
pps		−0.11	8.58	2.43	0.35	1.91	2.67
		(0.46)	(0.29)	(0.23)	(0.28)	(0.24)	(0.23)
spps		−0.01	3.48	0.91	−0.89	0.77	0.92
		(0.19)	(0.14)	(0.15)	(0.19)	(0.15)	(0.15)
spscx			7.41	0.38	−2.62	−0.47	0.33
			(0.24)	(0.21)	(0.30)	(0.23)	(0.21)
pscx			8.87	3.47	3.47	4.57	
			(0.26)	(0.35)	(0.35)	(0.29)	

[1]*Estimation bias rating: <0.20%, no bias; 0.20–1.00%, small bias; 1–5%, serious bias; >5%, very serious bias.*
[2]*YLR = YPI for SRS and RSRS.*
[3]*YBLU = YPR for SRS and RSRS.*
[4]*Not appropriate for pscx.*

ased estimators were found that were not also essentially unbiased for the linear populations.

Mean Square Error (tables 1 and 2)

In terms of mean square error, SRS and RSRS show a remarkably consistent pattern for the eight populations. With all estimators RSRS basically has lower mean square error than SRS, in some cases considerably lower (YRM and YLR in population 2, YRM in population 3, and YRM, YBLU, YPR, and YWR in population 7). In populations 5, 6, 7, and 8 YLR is the most efficient estimator. The other regression estimators show very little difference in performance for each population except that YWR is much less efficient than the others in population 7.

With pps sampling YBLU, YWR, and YPI tend to be the most efficient estimators throughout. YHT is equally efficient in populations 1, 2, and 5. It is much less efficient in populations 3, 4, 6, 8, and especially 7. YLR is much less efficient than YBLU, YWR, and YPI in populations 1, 2, and 3, and somewhat less efficient generally in the other populations. YPR is less efficient than YBLU, YPI, and YWR in populations 2, 3, 4, 6, and 7 and about equally efficient in the other populations.

With spps sampling YBLU, YWR, and YPI tend to be consistently the most efficient estimators and show very little difference among them. YLR is not efficient in populations 1, 2, and 3. YPR is inefficient in all populations. YHT is efficient in populations 1, 2, 3, and 4 but inefficient in 5, 6, 8, and especially in 7.

With spscx sampling YBLU, YPI, and YWR consistently give the most efficient estimators and there is little difference between the three estimators. YLR is quite inefficient for populations 1, 2, 3, and 7. YPR is generally inefficient in all populations.

With pscx sampling YWR is consistently the most efficient estimator except in populations 6 and 8 where

Table 2. Best mean square error sampling strategies for each population. (The values in parentheses have already been scaled for each population.)

Population					
1. pscx, YWR (0.64)	spscx, YWR (0.66)	spps, YWR (0.66)	spps, YHT (0.67)	spps, YBLU (0.67)	spscx, YPI (0.68)
2. pscx, YWR (0.42)	spscx, YPI (0.43)	spscx, YBLU (0.43)	spscx, YWR (0.46)	pps, YBLU (0.53)	spps, YHT (0.56)
3. spscs, YWR (4.13)	spscx,YPI (4.14)	spscx, YBLU (4.15)	pscx, YWR (4.22)	spps, YBLU (4.37)	spps, YPI (4.38)
4. pscx, YWR (6.91)	spps, YLR (7.15)	spps,YWR (7.28)	pscx, YBLU (7.28)	spps, YBLU (7.31)	pscx, YLR (7.33)
5. spps, YPI (6.09)	spps, YBLU (6.10)	spps, YLR (6.12)	spps, YWR (6.12)	spps, YHT (6.13)	spps, YPR (6.21)
6. RSRS, YLR (0.57)	RSRS, YBLU (0.58)	spps, YLR (0.58)	spps, YBLU (0.59)	spps, YWR (0.60)	spps, YPI (0.60)
7. RSRS, YLR (0.71)	RSRS, YBLU (0.79)	SRS, YLR (0.81)	spps, YPI (0.86)	spps, YWR (0.90)	spps, YBLU (0.90)
8. RSRS, YLR (0.13)	RSRS, YBLU (0.14)	RSRS, YPR (0.14)	SRS, YLR (0.14)	spps, YLR (0.14)	spps, YBLU YWR, YPI (0.15)

YLR is clearly best. YLR is quite inefficient in populations 1, 2, 3, 5, and to a lesser extent in 4. YBLU (= YPR for pscx) is only efficient in population 5.

The most efficient sampling strategies for each population are shown in table 2. The most efficient sampling designs generally are spps, spscx, and pscx for the populations with strong linear relationships and heterogeneous error structure (1–4). There is no estimator that is always best for these three sample selection methods but YWR is quite good in many instances. spps is clearly the best sample selection method with any regression estimator in population 5. In population 6 with a homogeneous error structure and populations 7 and 8 with weak linear relationships, RSRS is best with the simple linear regression estimator YLR, closely followed by RSRS with YBLU.

Conclusions on mean square error are as follows:

1. RSRS always has lower MSE than SRS for most estimators.
2. There is no consistently best sampling strategy for all populations, even for all populations with a strong linear y – x relationship.
3. Generally spps, spscx, and pscx with estimator YWR are good sampling strategies for the linear populations with heterogeneous error.
4. spps with any of the regression estimators and to a somewhat lesser extent RSRS are quite efficient for the linear populations with homogeneous error structure.
5. RSRS with YLR or YBLU is very efficient for the populations with weaker y – x relationships.

Variance Estimation Bias (table 3)

The variance estimation biases for classical, jackknife, and bootstrap variance estimators are given in table 3 for the eight populations.

Variance estimation bias can be summarized as follows.

1. None of the three variance estimators (classical, jackknife, bootstrap) dominates the others in terms of variance estimator bias for all eight populations, for all six sampling designs, or for all seven estimators considered.
2. In general, the bootstrap variance estimator tends to be the least biased for the cases studied. But the bootstrap variance estimator can be very seriously biased for YPI with pps and spscx for all eight populations.
3. The classical variance estimator of YWR has no bias for population 1 as expected. But for the other populations the bootstrap variance estimator tends to be the least biased.
4. The classical and the jackknife variance estimators of YHT for pps have small biases for all eight populations.
5. Sample selection methods spps, spscx, and pscx with estimator YWR tend to have the lowest mean square error. For these the following is true for variance estimation:
   a. spps with YWR tends to have reliable bootstrap variance estimates (≤8.58% for all 8 populations).
   b. spscx with YWR tends to have less reliable bootstrap variance estimates (≤20.89% for all 8 populations).
   c. pscx with YWR tends to have even less reliable bootstrap variance estimates (≤32.08% for all 8 populations).
6. Variance estimation with RSRS tends to be less biased than with SRS but sometimes is drastically worse.

Coverage Probability of 95% Confidence Interval (table 4)

As indicated earlier, jackknife variance estimates consistently overestimate the true variance of the estimated total. This explains why using the jackknife variance estimates, 43% of the time (113/264 cases) the coverage

Table 3.—Bias of classical, jackknife, and bootstrap variance estimators expressed as percent of the iterated variance for the seven classical estimators with the six sample selection methods with n = 20 and 10,000 iterations from each population (2,000 iterations for bootstrap variance estimates).[1]

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[4]
1. Simulated loblolly pine, volume N = 1795							
SRS	-6.87		-30.19	-6.93	-6.93	-1.16	-24.96
			{-23.4}[3]				
	(7.42)[4]		(27.17)	(12.79)	(13.61)	(9.31)	(27.17)
	[-12.88][5]		[-9.73]	[-7.32]	[-7.32]	[-2.56]	[-9.73]
RSRS	7.62		-18.40	-7.89	-7.89	-1.90	-18.15
			{-19.2}[3]				
	(23.99)		25.54	(12.09)	(12.94)	(9.21)	(25.54)
	[9.36]		[-3.75]	[-8.84]	[-8.85]	[-2.00]	[-3.75]
pps		1.56	42.92	-0.91	-11.48	0.56	55.14
		(0.43)	(19.16)	(20.90)	(36.84)	(24.47)	(20.11)
		[-42.35]	[4.24]	[12.56]	[-0.17]	[14.28]	[492.43]
spps		3.48	98.41	0.72	-12.09	1.41	52.86
		(62.35)	(13.39)	(7.65)	(15.19)	(7.74)	(12.78)
		[5.79]	[12.40]	[4.67]	[-8.47]	[5.80]	[90.52]
spscx			49.73	-3.29	-16.48	0.01	56.55
			(15.89)	(13.55)	(33.60)	(18.85)	(16.45)
			[6.08]	[7.71]	[-5.49]	[11.10]	[562.88]
pscx			48.45	-11.49	-11.49	0.56	
			(17.63)	(32.44)	(35.28)	(16.79)	
			[8.84]	[-1.06]	[-1.10]	[4.33]	
2. Loblolly pine, volume N = 1801							
SRS	-20.06		-22.20	-48.66	-48.66	-24.30	-16.80
			{-5.3}				
	(5.23)		(30.99)	(6.86)	(9.10)	(10.32)	(30.99)
	[19.36]		[-25.11]	[-43.49]	[-43.39]	[-19.69]	[-25.11]
RSRS	0.69		-28.98	-35.17	-35.17	-19.93	-28.77
			{-28.5}				
	(19.88)		(25.34)	(7.09)	(8.52)	(7.33)	(25.34)
	[-2.81]		[-14.75]	[-26.34]	[-26.27]	[-9.46]	[-14.75]
pps		4.19	12.54	34.43	-42.11	22.81	130.87
		(3.03)	(25.24)	(26.45)	(25.55)	(27.94)	(36.98)
		[-9.68]	[-11.05]	[29.00]	[-41.35]	[19.07]	[1022.77]
spps		3.80	269.44	-18.51	-66.16	-24.30	90.24
		(26.11)	(63.88)	(9.97)	(11.31)	(9.58)	(32.84)
		[1.04]	[39.38]	[-2.41]	[-58.27]	[-8.58]	[93.48]
spscx			68.43	35.46	-41.59	26.41	194.51
			(87.78)	(27.33)	(68.37)	(36.83)	(53.93)
			[14.93]	[27.31]	[-45.24]	[17.75]	[1373.74]
pscx			112.93	-25.98	-25.98	9.60	
			(64.39)	(44.35)	(49.19)	(23.15)	
			[32.09]	[-24.19]	[-24.02]	[11.80]	
3. *Pinus radiata*, volume N = 2761							
SRS	-5.07		-23.32	-18.56	-18.56	-10.07	-18.36
			{-10.0}				
	(4.36)		(23.10)	(11.08)	(11.83)	(10.29)	(23.10)
	[-9.53]		[-11.35]	[-15.60]	[-15.58]	[-7.28]	[-1.34]
RSRS	11.69		-15.74	-17.35	-17.35	-10.88	-5.32
			{-14.9}				
	(21.75)		(22.64)	(10.55)	(11.23)	(8.97)	(22.64)
	[5.74]		[-2.65]	[-9.41]	[-9.40]	[-2.56]	[-2.65]
pps		-0.74	50.66	7.08	-5.93	6.41	51.38
		(-1.46)	(14.34)	(11.28)	(17.63)	(12.69)	(12.85)
		[-29.38]	[0.50]	[3.30]	[-9.06]	[2.55]	[234.05]
spps		-0.21	95.42	-6.64	-20.02	-7.09	38.77
		(40.69)	(15.96)	(6.76)	(12.13)	(6.96)	(12.81)
		[-2.55]	[9.71]	[-5.56]	[-19.17]	[-6.23]	[35.61]
spscx			77.79	6.21	-7.35	6.77	55.05
			(22.45)	(9.88)	(22.12)	(12.72)	(13.62)
			[9.94]	[2.38]	[-11.38]	[2.72]	[264.77]
pscx			116.94	0.15	0.15	15.96	
			(51.04)	(39.37)	(42.02)	(21.42)	
			[36.38]	[-11.42]	[-11.37]	[5.05]	

Table 3.—Continued

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[4]
4. Southern, remeasured N = 275							
SRS	1.61		-8.20	60.18	60.18	57.95	-2.76
			{5.1}				
	(3.41)		(10.79)	(13.08)	(13.50)	(11.65)	(10.78)
	[4.96]		[-3.62]	[1.19]	[1.06]	[1.27]	[-3.62]
RSRS	-0.15		-7.49	53.51	53.51	54.62	-6.51
			{-13.7}				
	(2.35)		(6.57)	(6.36)	(6.80)	(6.37)	(6.57)
	[8.78]		[2.43]	[9.46]	[9.33]	[9.57]	[2.43]
pps		5.77	18.22	-14.92	-25.06	-17.43	-0.78
		(-1.93)	(19.10)	(15.27)	(19.48)	(16.32)	(13.77)
		[-39.79]	[28.10]	[154.66]	[104.67]	[10.89]	[85.29]
spps		7.80	29.33	3.80	-6.06	4.23	7.56
		(84.01)	(10.76)	(12.64)	(18.72)	(13.49)	(10.99)
		[3.55]	[8.56]	[38.09]	[22.31]	[3.41]	[23.00]
spscx			31.79	-23.54	-36.24	-25.85	-9.99
			(16.18)	(19.84)	(28.29)	(22.79)	(17.92)
			[14.94]	[67.44]	[34.93]	[-2.68]	[67.93]
pscx			51.63	-4.64	-4.64	-2.15	
			(15.36)	(50.96)	(52.05)	(38.25)	
			[30.39]	[33.73]	[33.83]	[32.08]	
5. Pine Creek, diameter N = 2748							
SRS	2.66		-6.02	27.20	27.20	27.66	1.20
			{35.5}				
	(4.11)		(16.53)	(15.93)	(16.03)	(14.44)	(16.53)
	[-6.76]		[7.94]	[10.02]	[10.02]	[10.33]	[7.94]
RSRS	6.22		-3.38	26.15	26.15	25.23	-1.58
			{-0.24}				
	(6.92)		(10.28)	(11.30)	(11.35)	(9.87)	(10.28)
	[8.80]		[12.62]	[16.02]	[16.02]	[15.73]	[12.62]
pps		2.04	-9.26	-7.22	-9.68	-7.54	-7.61
		(1.30)	(6.31)	(7.51)	(8.96)	(7.78)	(7.18)
		[-19.31]	[-0.13]	[12.37]	[8.56]	[-2.87]	[13.01]
spps		0.82	3.07	10.75	8.94	10.38	1.84
		(24.97)	(8.26)	(9.16)	(11.48)	(9.59)	(8.45)
		[-0.67]	[-0.15]	[5.69]	[4.65]	[0.62]	[3.61]
spscx			-3.30	-5.88	-6.81	-5.69	-5.99
			(5.88)	(8.01)	(11.63)	(8.67)	(7.86)
			[3.04]	[21.64]	[20.71]	[3.88]	[20.25]
pscx			-5.10	-7.92	-7.92	-7.04	
			(5.68)	(10.55)	(10.75)	(8.39)	
			[1.53]	[1.50]	[1.50]	[0.92]	
6. Alabama, remeasured N = 1905							
SRS	2.09		-7.24	168.89	168.89	116.20	-1.26
			{3.7}				
	(6.49)		(12.68)	(25.24)	(25.43)	(18.41)	(12.68)
	[2.66]		[-9.42]	[9.54]	[9.46]	[-0.56]	[-9.42]
RSRS	57.99		-3.87	187.35	187.35	136.01	-3.21
			{-3.7}				
	(63.59)		(9.33)	(21.08)	(21.41)	(18.04)	(9.33)
	[1.58]		[-3.54]	[15.47]	[15.40]	[3.93]	[-3.54]
pps		0.13	-10.49	-34.35	-46.41	-36.78	-27.85
		(-0.93)	(8.42)	(15.83)	(20.72)	(16.93)	(12.56)
		[-68.32]	[2.68]	[328.59]	[246.95]	[-13.41]	[54.23]
spps		2.86	12.08	19.13	-2.13	18.79	-4.60
		(346.01)	(8.14)	(17.41)	(25.73)	(17.35)	(11.39)
		[-25.27]	[-5.33]	[77.27]	[46.60]	[-5.79]	[13.45]
spscx			-1.84	-39.76	-57.21	-43.86	-33.87
			(9.99)	(26.07)	(36.92)	(29.81)	(20.40)
			[1.85]	[55.44]	[11.50]	[-15.57]	[52.48]
pscx			-7.91	-60.75	-60.75	-49.07	
			(7.27)	(27.55)	(29.53)	(24.89)	
			[-0.47]	[-36.86]	[-36.81]	[-21.75]	

Table 3.—Continued

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[4]
7. *Pinus radiata*, bark N = 5142							
SRS	6.12		−13.98	3246.98	3246.98	662.93	−7.87
		2089.07	{−7.6}				
	(19.12)		(14.74)	(87.69)	(82.77)	(41.56)	(14.74)
	[34.16]		[6.61]	[344.86]	[342.26]	[−21.72]	[6.61]
RSRS	352.30		−0.46	11556.6	11556.6	882.49	−0.16
			{−0.4}				
	(381.38)		(13.93)	(245.81)	(241.30)	(54.53)	(13.93)
	[534.32]		[16.21]	[1979.85]	[−10.98]	[16.21]	
pps		0.05	−56.11	−64.23	−71.67	−65.59	−56.72
		(−0.34)	(10.35)	(45.50)	(54.13)	(48.16)	(36.84)
		[−97.88]	[−2.36]	[82366.5]	[65204.5]	[−7.31]	[87.66]
spps		0.71	7.24	42.87	−18.52	45.42	−10.18
		(2089.07)	(27.64)	(36.76)	(51.27)	(36.73)	(20.90)
		[−72.22]	[12.98]	[80236.5]	[45251.2]	[2.79]	[93.11]
spscx			−33.31	−73.83	−82.78	−74.84	−66.13
			(43.33)	(47.96)	(58.50)	(53.34)	(53.70)
			[35.77]	[107.22]	[39.75]	[−4.52]	[196.86]
pscx			−34.55	−85.70	−85.70	−78.70	
			(45.65)	(79.40)	(83.06)	(78.54)	
			[38.62]	[−33.19]	[−33.16]	[−11.66]	
8. New York, remeasured N = 622							
SRS	−1.95		−6.10	236.20	236.20	171.01	−0.27
			{11.5}				
	(2.83)		(13.51)	(27.91)	(28.13)	(27.16)	(13.51)
	[−1.82]		[−1.56]	[6.10]	[5.92]	[1.85]	[−1.56]
RSRS	49.95		−4.97	271.07	271.07	186.33	−4.20
			{−3.7}				
	(53.64)		(6.37)	(23.26)	(23.59)	(20.59)	(6.37)
	[58.05]		[−4.71]	[16.30]	[16.01]	[7.69]	[−4.71]
pps		0.99	−22.17	−37.38	−51.61	−41.48	−32.68
		(−2.26)	(15.25)	(22.37)	(24.08)	(22.55)	(18.62)
		[−72.16]	[8.57]	[530.91]	[361.09]	[−6.87]	[58.90]
spps		2.22	12.63	11.04	−15.25	10.86	−10.87
		(407.40)	(12.38)	(20.92)	(29.24)	(21.96)	(13.98)
		[−18.64]	[3.12]	[91.86]	[45.22]	[−2.59]	[15.19]
spscx			−6.29	−46.51	−62.86	−51.45	−38.82
			(25.58)	(32.35)	(43.65)	(35.50)	(32.49)
			[11.07]	[42.34]	[−3.38]	[−20.09]	[55.10]
pscx			−8.35	−64.97	−64.97	−55.42	
			(17.84)	(40.46)	(42.42)	(38.58)	
			[9.17]	[−32.85]	[−32.69]	[−18.47]	

[1] *Variance bias ratings: <5%, no bias; 5–10%, small bias; 10–20%, moderate bias; 20–50%, serious bias; >50%, very serious bias.*

[2] *Not appropriate for pscx.*

[3] *$v_H(\hat{Y}_{\ell r})$ is only shown for SRS and RSRS in braces.*

[4] *The values in parentheses are the biases of jackknife variance estimates.*

[5] *The values in brackets are the biases of bootstrap variance estimates.*

probabilities fall in the interval (0.94, 0.96) as opposed to only 26% for the classical (67/264) and 33% for the bootstrap (86/264). These percentages increase to 63%, 46%, and 49%, respectively if we consider the interval ≥0.94 to be acceptable.

Coverage probabilities can be summarized as follows. With this sample size (n = 20) one can expect unacceptable coverage probabilities for at least some populations with any sampling strategy because the sample size is so small and several populations are highly skewed; this is what basically happened. The only exceptions are pps and spps sampling with YPI using the bootstrap variance estimator and spps with YHT using the jackknife variance estimator. Our principal interest with coverage probabilities is to see how sampling strategies performed that had either low estimation bias and/or low mean square error and/or low variance estimation bias.

Both pps and spps with YHT provide unbiased estimates of the population total and essentially unbiased

Table 4.—Percentage of coverage probability of 95% confidence interval that includes true population total for the seven classical, jackknife, and bootstrap estimators with the six sample selection methods with n = 20 and 10,000 iterations from each population (2,000 iterations for bootstrapping).

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[1]
1. Simulated loblolly pine, volume N = 1795							
SRS	92.1		84.6	94.1	94.1	94.8	86.3
	(93.0)[2]		(92.3)	(93.2)	(93.2)	(93.9)	(92.3)
	[90.4][3]		[90.4]	[92.1]	[92.1]	[92.1]	[90.4]
RSRS	93.9		89.5	94.0	94.0	94.8	89.5
	(94.5)		(92.8)	(93.2)	(93.2)	(94.1)	(92.8)
	[93.9]		[91.5]	[91.5]	[91.5]	[92.7]	[91.5]
pps		95.0	97.9	94.5	93.4	94.8	97.8
		(94.9)	(97.6)	(95.0)	(94.4)	(94.9)	(95.3)
		[86.0]	[96.0]	[94.2]	[92.9]	[93.9]	[99.8]
spps		93.9	98.9	95.0	93.6	95.0	97.5
		(98.2)	(96.5)	(95.3)	(94.6)	(95.2)	(95.6)
		[93.2]	[96.1]	[94.3]	[92.8]	[94.3]	[98.5]
spscx			98.4	94.8	93.1	95.2	98.3
			(97.8)	(95.8)	(95.1)	(95.9)	(96.1)
			[96.2]	[94.7]	[93.3]	[94.9]	[100.0]
pscx			98.4	93.5	93.5	94.9	
			(97.8)	(94.7)	(94.8)	(95.3)	
			[97.2]	[93.0]	[93.1]	[95.3]	
2. Loblolly pine, volume N = 1801							
SRS	89.7		82.4	86.6	86.6	91.3	84.1
	(90.8)		(92.1)	(92.9)	(93.0)	(93.5)	(92.1)
	[89.9]		[88.4]	[87.8]	[87.8]	[91.0]	[88.4]
RSRS	91.6		86.5	88.8	88.8	91.1	86.5
	(92.4)		(91.5)	(92.3)	(92.3)	(92.9)	(91.5)
	[92.2]		[90.7]	[89.9]	[89.9]	[91.7]	[90.7]
pps		95.3	97.3	96.9	86.8	96.4	98.7
		(95.1)	(98.2)	(95.7)	(95.7)	(95.8)	(96.5)
		[91.9]	[96.9]	[94.8]	[86.7]	[94.8]	[99.9]
spps		92.2	99.3	92.6	79.4	82.1	97.7
		(97.5)	(96.5)	(95.2)	(94.8)	(95.2)	(95.9)
		[89.8]	[96.2]	[93.7]	[83.1]	[93.5]	[97.7]
spscx			99.5	96.9	88.0	96.6	99.4
			(99.1)	(96.5)	(97.7)	(96.9)	(97.1)
			[97.6]	[95.6]	[86.0]	[95.4]	[100.0]
pscx			99.0	92.7	92.7	95.8	
			(97.6)	(97.0)	(97.2)	(95.8)	
			[96.3]	[92.0]	[92.2]	[94.6]	
3. *Pinus radiata*, volume N = 2761							
SRS	93.6		91.4	96.2	96.2	96.3	92.2
	(93.7)		(93.2)	(94.1)	(94.1)	(94.4)	(93.2)
	[92.9]		[92.8]	[93.4]	[93.3]	[93.8]	[92.8]
RSRS	93.3		91.9	95.9	95.9	96.1	92.0
	(93.3)		(92.9)	(93.3)	(93.3)	(93.9)	(92.9)
	[94.7]		[92.5]	[93.5]	[93.5]	[93.5]	[92.5]
pps		92.9	93.5	91.1	89.8	90.7	92.8
		(92.1)	(94.2)	(92.4)	(92.1)	(92.4)	(92.8)
		[90.2]	[94.4]	[96.3]	[96.0]	[93.2]	[97.5]
spps		92.7	94.9	93.5	93.0	93.5	94.0
		(95.3)	(94.3)	(94.0)	(94.1)	(94.1)	(94.1)
		[92.4]	[93.4]	[94.9]	[94.2]	[92.8]	[94.9]
spscx			94.6	90.1	87.3	89.8	92.4
			(94.8)	(92.9)	(92.9)	(92.9)	(93.0)
			[93.5]	[96.8]	[95.9]	[92.4]	[97.7]
pscx			95.1	88.9	88.9	89.7	
			(94.1)	(92.0)	(92.0)	(92.2)	
			[94.8]	[92.1]	[92.1]	[92.6]	
4. Southern, remeasured N = 275							
SRS	93.1		88.2	92.4	92.4	93.6	89.4
	(93.8)		(93.9)	(94.5)	(94.5)	(94.7)	(93.9)
	[94.2]		[92.6]	[92.3]	[92.3]	[93.8]	[92.6]
RSRS	95.6		91.1	93.1	93.1	93.8	91.2
	(96.1)		(94.2)	(94.6)	(94.6)	(94.9)	(94.2)
	[93.4]		[93.5]	[93.4]	[93.4]	[94.2]	[93.5]

Table 4.—Continued

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[1]
pps		95.0	98.1	95.6	93.9	95.5	97.6
		(95.0)	(97.1)	(95.4)	(95.1)	(95.3)	(95.7)
		[85.8]	[96.4]	[94.6]	[92.6]	[94.2]	[99.7]
spps		93.3	98.7	94.2	92.2	94.2	97.3
		(98.0)	(96.4)	(95.6)	(95.5)	(95.6)	(95.9)
		[92.0]	[95.3]	[93.6]	[91.7]	[93.5]	[96.9]
spscx			98.9	95.9	93.9	95.7	98.2
			(97.9)	(95.9)	(95.9)	(96.0)	(96.2)
			[96.9]	[94.4]	[92.7]	[94.5]	[99.8]
pscx			99.0	93.7	93.7	96.0	
			(98.0)	(96.0)	(96.1)	(96.1)	
			[96.7]	[92.4]	[92.4]	[94.6]	
5. Pine Creek, diameter N = 2748							
SRS	94.6		93.8	96.0	96.0	96.0	94.8
	(94.7)		(95.2)	(95.4)	(95.3)	(95.3)	(95.2)
	[94.0]		[93.9]	[94.2]	[94.2]	[94.2]	[93.9]
RSRS	95.0		94.1	96.3	96.3	96.3	94.2
	(95.0)		(95.0)	(95.2)	(95.2)	(95.3)	(95.0)
	[94.7]		[95.1]	[95.2]	[95.2]	[95.4]	[95.1]
pps		93.8	94.0	93.1	92.4	93.0	93.4
		(93.8)	(95.2)	(94.0)	(93.6)	(94.0)	(94.2)
		[90.6]	[94.5]	[94.8]	[93.9]	[92.8]	[95.4]
spps		93.0	95.1	95.2	94.7	95.1	94.6
		(95.9)	(95.4)	(95.2)	(95.1)	(95.2)	(95.3)
		[92.8]	[94.2]	[94.6]	[94.3]	[94.0]	[94.3]
spscx			94.4	93.4	92.9	93.5	93.7
			(95.2)	(94.8)	(94.3)	(94.6)	(94.8)
			[94.4]	[95.1]	[94.7]	[93.3]	[95.4]
pscx			94.1	92.7	92.7	93.1	
			(95.0)	(94.0)	(94.0)	(94.4)	
			[94.6]	[93.4]	[93.4]	[93.7]	
6. Alabama, remeasured N = 1905							
SRS	95.2		94.2	98.2	98.2	98.4	95.0
	(95.4)		(95.8)	(96.5)	(96.5)	(96.9)	(95.8)
	[95.7]		[93.1]	[94.0]	[94.0]	[95.1]	[93.1]
RSRS	98.2		94.6	98.8	98.8	98.6	84.7
	(98.3)		(95.4)	(96.2)	(96.1)	(96.4)	(95.4)
	[97.8]		[94.1]	[95.8]	[95.8]	[94.9]	[94.1]
pps		88.9	93.5	88.9	85.4	88.1	91.1
		(88.7)	(95.6)	(93.1)	(92.2)	(92.9)	(93.3)
		[75.0]	[94.2]	[96.0]	[95.4]	[91.3]	[97.8]
spps		92.8	96.0	95.5	93.6	95.5	94.5
		(98.5)	(95.8)	(95.5)	(95.0)	(95.5)	(95.5)
		[91.5]	[94.0]	[95.7]	[94.4]	[93.6]	[95.4]
spscx			94.2	87.5	81.0	86.3	89.5
			(95.6)	(92.8)	(91.8)	(92.6)	(93.0)
			[94.8]	[97.6]	[95.5]	[90.9]	[97.4]
pscx			93.8	81.5	81.5	85.4	
			(95.5)	(92.1)	(92.1)	(93.0)	
			[94.5]	[88.2]	[88.2]	[90.5]	
7. *Pinus radiata*, bark N = 5142							
SRS	94.7		92.2	99.8	99.8	99.7	93.5
	(94.7)		(94.8)	(97.8)	(97.9)	(98.9)	(94.8)
	[95.2]		[93.9]	[96.6]	[96.6]	[97.2]	[93.9]
RSRS	99.9		94.6	100.0	100.0	99.9	94.7
	(99.9)		(95.8)	(98.3)	(98.3)	(99.3)	(95.8)
	[100.0]		[95.5]	[98.7]	[98.7]	[98.5]	
pps		68.1	67.3	79.1	73.0	77.9	83.0
		(68.0)	(88.0)	(91.2)	(89.5)	(90.8)	(91.4)
		[37.6]	[84.7]	[93.1]	[92.5]	[90.2]	[96.3]
spps		79.5	95.1	95.7	90.1	95.7	94.2
		(100.0)	(96.2)	(95.3)	(92.7)	(95.4)	(95.7)
		[77.7]	[94.9]	[97.1]	[93.0]	[93.6]	[98.8]

Table 4.—Continued

Population Sampling design	YRM	YHT	YLR[2]	YBLU	YPR[3]	YWR	YPI[1]
spscx			69.9	71.2	62.2	70.1	78.8
			(88.8)	(90.1)	(89.8)	(90.0)	(91.0)
			[86.7]	[99.5]	[98.7]	[92.7]	[99.7]
pscx			73.2	57.1	57.1	68.1	
			(91.5)	(89.7)	(89.8)	(91.7)	
			[89.7]	[90.4]	[90.5]	[93.4]	
8. New York, remeasured N = 622							
SRS	95.3		93.9	98.0	98.0	98.4	94.6
	(95.6)		(95.5)	(96.3)	(96.3)	(96.8)	(95.5)
	[94.8]		[93.8]	[94.6]	[94.5]	[95.5]	[93.8]
RSRS	98.0		94.6	99.1	99.1	99.1	94.6
	(98.1)		(95.6)	(96.6)	(96.6)	(97.0)	(95.6)
	[98.2]		[94.4]	[95.7]	[95.7]	[96.0]	[94.4]
pps		88.8	89.1	88.5	84.6	87.8	90.3
		(88.5)	(94.6)	(93.2)	(92.7)	(93.1)	(93.6)
		[79.4]	[94.0]	[95.9]	[95.0]	[93.2]	[98.4]
spps		92.1	95.8	94.7	91.4	94.6	94.0
		(98.3)	(96.0)	(95.4)	(95.0)	(95.6)	(95.7)
		[91.7]	[94.4]	[95.1]	[93.5]	[93.8]	[95.7]
spscx			92.0	85.5	76.4	83.4	88.6
			(95.4)	(93.0)	(93.0)	(92.9)	(93.5)
			[93.9]	[95.3]	[93.1]	[90.4]	[97.8]
pscx			89.8	75.5	75.5	80.9	
			(93.4)	(93.6)	(93.7)	(93.3)	
			[92.3]	[88.4]	[88.4]	[91.1]	

[1]*YLR = YPI for SRS and RSRS.*

[2]*The values in parentheses are the percentages of coverage probabilities for the jackknife estimators.*

[3]*The values in brackets are the percentages of coverages for the bootstrap estimators.*

variance estimates for all populations. spps with YHT using the jackknife variance estimates provides coverage probabilities larger than 0.95 in all populations.

YLR is essentially unbiased for SRS and RSRS in all eight populations. Satisfactory coverage probabilities are only found in populations 3 and 5–8, particularly with RSRS, the use of the jackknife variance estimator giving consistently the highest coverage probabilities.

YRM is essentially unbiased for SRS and RSRS in the first six populations where a linear relationship exists between the variables. But satisfactory coverage probabilities are generally only found in populations 3 and 5–8.

spps sampling with estimator YWR tends to be quite efficient and tends to have reliable bootstrap variance estimates. But with this sampling strategy the classical and jackknife variance estimates yield satisfactory coverage probabilities in basically all populations especially the latter. spscx sampling with estimator YWR is quite efficient too. But coverage probabilities tend to be acceptable only in populations 1, 2, 3, and 5.

Summary

With the large variety in populations, estimators, sample designs, and criteria it is difficult to identify sampling strategies that are clearly best. Traditionally the main criteria are negligible estimation bias combined with small mean square error. In that regard spps and spscx with estimator YWR are good sampling strategies, particularly for the six populations with linear relationships. pscx with YWR is quite efficient too across populations but suffers from estimation bias problems in several cases. pps and spps with YHT have the advantage of being unbiased for all populations as well as having essentially unbiased classical and jackknife (for pps only) variance estimators for all populations. But YHT is quite inefficient in several populations.

RSRS very consistently has lower mean square error than SRS with all estimators. There was no obvious pattern on whether SRS or RSRS was better in terms of estimation bias.

In terms of these most efficient estimators discussed above, spps with YWR tends to have reliable bootstrap variance estimates. Those for spscx with YWR tend to be less reliable. Interestingly, use of the classical and jackknife variance estimates but not the bootstrap give satisfactory coverage probabilities in most cases for spps with YWR.

Literature Cited

Cochran, W. G. 1977. Sampling techniques. 3d ed. New York: Wiley and Sons. 428 p.

Cumberland, W. G.; Royall, R. M. 1981. Prediction models and unequal probability sampling. Journal of the Royal Statistical Society B. 43(3): 353–367.

Efron, B. 1982. The jackknife, the bootstrap, and other resampling plans. SIAM, Monograph #38. CBMS - NSF.

Herson, J. 1976. An investigation of relative efficiency of least-squares prediction to conventional probability sampling plans. Journal of the American Statistical Association. 71: 700–703.

Iachan, R. 1985. Robust designs for ratio and regression estimation. Journal of Statistical Planning and Inference. 11: 149–161.

Karmel, T. S.; Jain, M. 1987. Comparison of purposive and random sampling schemes for estimating capital expenditure. Journal of the American Statistical Association. 82: 52–57.

Lewis, N. B.; McIntyre, G. A.; Leech, J. W. 1973. Regional volume table for *Pinus radiata* in South Australia. Metric edition. Bulletin 20. South Australia: Woods and Forests Department. 64 p.

Little, R. J. A. 1983. Estimating a finite population mean from unequal probability samples. Journal of the American Statistical Association. 78: 596–604.

McClure, J. P.; Anderson, J.; Schreuder, H. T. 1987. A comparison of regional and site-specific volume estimation equations. Res. Pap. SE-264. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 9 p.

McClure, J. P.; Schreuder, H. T.; Wilson, R. L. 1983. A comparison of volume table equations for loblolly pine and white oak. Res. Pap. SE-240. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 8 p.

Rao, J. N. K.; Wu, C. F. J. 1988. Resampling inference with complex survey data. Journal of the American Statistical Association. 83: 231–241.

Royall, R. M.; Cumberland, W. G. 1978. Variance estimation in finite population sampling. Journal of the American Statistical Association. 73: 351–358.

Royall, R. M.; Cumberland, W. G. 1981a. An empirical study of the ratio estimator and estimators of its variance (with discussion). Journal of the American Statistical Association. 76: 66–88.

Royall, R. M.; Cumberland, W. G. 1981b. The finite-population linear regression estimators and estimators of its variance—an empirical study. Journal of the American Statistical Association. 76: 924–930.

Royall, R. M.; Herson, J. 1973a. Robust estimation in finite populations. I. Journal of the American Statistical Society. 68: 880–889.

Royall, R. M.; Herson, J. 1973b. Robust estimation in finite populations. II. stratification on a size variable. Journal of the American Statistical Association. 68: 890–893.

Särndal, C. E. 1980. On II-inverse weighting versus BLU weighting in probability sampling. Biometrika. 67(3): 639–650.

Särndal, C. E. 1982. Implications of survey design for generalized regression estimation of linear functions. Journal of Statistical Planning and Inference. 6: 155–170.

Schreuder, H. T. 1986. Model-dependent vs. design-dependent procedures in timber sale and updating surveys. In: Proceedings, SAF national meeting; 1986 October; Birmingham, AL: 74–76.

Schreuder, H. T.; Anderson, J. 1984. Variance estimation for volume when D^2H is the covariate in regression. Canadian Journal of Forest Research. 14(6): 818–821.

Schreuder, H. T.; Li, H. G.; Hazard, J.. 1987. pps and random sampling estimation using some regression and ratio estimators for underlying linear and curvilinear models. Forest Science. 33(4): 997–1009.

Schreuder, H. T.; Thomas, C. E. 1985. Efficient sampling techniques for timber sale surveys and inventory updates. Forest Science. 31(4): 857–866.

Schreuder, H. T.; Wood, G. B. 1986. The choice between design-dependent and model-dependent sampling. Canadian Journal of Forest Research. 16(2): 260–265.

Spurr, S. H. 1952. Forest inventory. New York: The Ronald Press Company. 476 p.

Wood, G. B.; Schreuder, H. T. 1986. Implementing point-Poisson and point-model based sampling in forest inventory. Forest Ecology and Management. 14(2): 141–156.

Rocky
Mountains

Southwest

Great
Plains

U.S. Department of Agriculture
Forest Service

Rocky Mountain Forest and Range Experiment Station

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

RESEARCH FOCUS

Research programs at the Rocky Mountain Station are coordinated with area universities and with other institutions. Many studies are conducted on a cooperative basis to accelerate solutions to problems involving range, water, wildlife and fish habitat, human and community development, timber, recreation, protection, and multiresource evaluation.

RESEARCH LOCATIONS

Research Work Units of the Rocky Mountain Station are operated in cooperation with universities in the following cities:

Albuquerque, New Mexico
Flagstaff, Arizona
Fort Collins, Colorado*
Laramie, Wyoming
Lincoln, Nebraska
Rapid City, South Dakota
Tempe, Arizona

*Station Headquarters: 240 W. Prospect Rd., Fort Collins, CO 80526